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ACTIVE INTELLIGENT CONTROL OF VIBRATION OF FLEXIBLE PLATE STRUCTURES

by

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ABSTRACT

The development of intelligent control approaches for vibration reduction of flexible plate structures are investigated and reported in this thesis. In this work active intelligent control comprises a set of control techniques based on particle swarm optimisation (PSO), real coded genetic algorithm (RCGA) and artificial immune system (AIS). A traditional method of recursive least squares (RLS) is investigated as a comparison to the proposed intelligent techniques. The aim of this work is to assess the potential applicability of the intelligent techniques in the active vibration control (AVC) of flexible structures. A simulation environment characterising the dynamic behaviour of a flexible plate structure is developed using finite difference methods and state-space formulation. This is realised within Matlab/SIMULINK as a testbed for verification of control designs. The plate is subjected to different disturbance signals and linear parametric models characterising the input/output dynamic behaviour of the plate, between two measurement points, is developed using RLS, RCGA, PSO and AIS algorithms. It is demonstrated through time-domain and frequency-domain analysis and tests that the RCGA, PSO and AIS approaches perform very well in modelling of the flexible plate. The modelling approach is extended to a model-based AVC strategy using the principle of wave interference. The approach is first realised within a single-input single-output (SISO) control configuration with RLS, RCGA, PSO and AIS algorithms. Tests with various disturbance signals show that good vibration reduction is achieved with the developed model-based SISO-AVC algorithms. The approach is then realised within a single-input multi-output (SIMO) control configuration, and exemplified in vibration control tests with two control sources. It is shown that higher levels of vibration can be achieved with model-based SIMO-AVC algorithms as compared to those model-based SISO-AVC algorithms. A further strategy based on non-model based control is developed for vibration reduction in flexible structures. The approach is realised within SISO and SIMO AVC structures using RCGA, PSO and AIS algorithms. These are implemented on the flexible plate structure with various disturbance signals. It is demonstrated that the non-model based AVC algorithms perform comparatively similar to their model-based AVC counterparts in terms of amount of vibration reduction. However, the non-model based AVC algorithms are faster than their model-based AVC counterparts.

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Chapter 1

Introduction

1.1 Background

Problems of flexible structures are extensively of engineering interest. Plate in the category of flexible structure is normally considered as large in size and small in thickness thus it exhibits dense modes, time-varying characteristics and non-linearity. Investigation into the dynamics of a flexible structure in an experimental setting may be costly. Simulations give benefits to illustrate the simplicity of implementation and the power of proposed algorithms in difficult realistic situations. Therefore, many investigators have studied dynamic characterisation of flexible structures with numerical methods and in simulation environment (Chen, 2001; De Gersem et al., 2005; Guennam and Luccioni, 2009). Different methods have been developed to study the dynamic equations of the plate analytically. These include the finite difference and differential transformation methods (Yeh et al., 2006), Galerkin's method in coupling with boundary element method (Panzeca et al., 2009), finite element method (Sheng and Ye, 2002) and the finite strip method (Attallah et al., 2007)

Research interest is more focused on flexible structures than rigid structures as they are capable of being operated at high speeds and handling of larger payloads with the same actuator capabilities (Tokhi, 2004). Flexible structures also offer several other advantages including light weight, lower energy consumption, safer operation due to reduced inertia, smaller actuator requirement, low mounting strength requirement, low rigidity requirement and less bulky design. These advantages lead to extensive usage of plate in various applications such as space vehicle, automotive industry and tall buildings.

The plate may resemble some of the panel cover of a space vehicle during launch. (Falangas et al., 1994). Flexible spacecraft in space is often subjected to random disturbances arising from various sources such as meteorite collision, radiation in solar pressure and variations in magnetic field as well as on-board disturbances (sloshing of liquid fuel, motor interruption). These disturbances induce randomly distributed forces and random torque and affect system stability (Hu and Ma, 2006). Hostile environment loadings, strong winds, earthquake motion induce severe vibration in bridge towers and tall buildings (Kar et al., 2000). Engineering applications of flexible structure include a wide range such as in skyscrapers and bridges in civil engineering applications; propellers, aircraft fuselage and wings, satellite solar panels, and helicopter blades in aerospace structures; and turbo generator shafts, engines, gas turbine rotors, and electric transformer cores in electromechanical systems (Tokhi and Hossain, 1994). Resonance may occur in a vibrating flexible plate when it is subjected to disturbances and that can cause damage to the structure. Thus it is essential to suppress the vibration with passive or active control techniques.

The most commonly applied vibration control techniques are based on the use of passive technologies. Passive control is a traditional method that consists of mounting passive material on the structure. The majority of application based on passive method use viscoelastic materials such as viscous dampers (dashpots), tuned-mass dampers, dynamic absorbers, shunted piezoceramic dampers, and magnetic dampers. This method is valuable in reducing vibration at high frequencies or in small frequency ranges but has limitation in added weight and poor low-frequency performance. To resolve these problems, active vibration control is realised as a potential solution. The work presented in this thesis focuses on development intelligent active vibration control algorithm with application to flexible plate structures.

1.2 Intelligent control

Intelligent control is defined as the ability to comprehend reason and learn. In an intelligent system, a collection of typical answers can be given allowing the system clear and easy to be used. When there is change of human operators, tasks and timeframe, an 'intelligent' system capable enough to analyze and interpret the result based on rules, objects and algorithms. Generally this narrows the gap of knowledge background among the users.

An intelligent controller mimics human intelligence such as in adaptation and learning, planning under large uncertainty, copying with large amounts of data, in order to effectively control complex processes. In addition, intelligence requires the ability to sense the environment, to make decisions and to control action. Albus in (Antsaklis and Passino, 1993) addressed that higher levels of intelligence may include the ability to recognise objects and events, to represent knowledge in a world model and to reason about and plan for the future. In other words, intelligence provides the capacity to perceive and understand, to choose wisely, and to act successfully under a large variety of circumstances so as to survive, prosper, and reproduce in a complex and often hostile environment.

Its levels can be perceived in a degree of adaptation and learning, autonomy and intelligence and also structural and hierarchies. Specifically in adaptation and learning, an intelligent system is needed to learn so that it can adapt a broad range of unexpected changes. While for the aspect of autonomy and intelligence, fixed controller is less autonomous than adaptive controller as fixed controller has lower certainty level than adaptive controller. In achieving the same goal, functional structure and hierarchies of range are required to be 'sparse', to manage with complexity of the system. In addition to the features above, the intelligent control can address control problems that cannot be formulated in the language of conventional control. For instance to panacea problem in a rolling steel mill, space structure, chemical processes, etc. (Antsaklis, 1999)

The heuristic method is also outlined in tuning parameters, objective function design in running operated system, necessity of learning from past experience and planning control action. The work considered that the performance of the operators is higher if the following can be achieved; increase speed of response, relieve the operator from routine tasks and hazard protection. To achieve this, high level decision making techniques for reasoning under uncertainty and taking actions must be utilised. These techniques, if used by humans, may be attributed to intelligent behaviour. Hence, one way to achieve high degree of autonomy is to utilise high level decision making techniques, intelligent methods, in the autonomous controller. Therefore, when autonomy becomes objective, the one way to achieve it is by using intelligent controller.

The use of active intelligent control for vibration reduction in many applications has become essential interest. Some papers use patches of piezoeelectric actuator to excite the plate structure. Yaman et al. (2002) used PZT patches to excite the plate and use finite element model within ANSYS software to find the optimal locations and configuration of the actuator to achieve the plate modes (Yaman et al., 2002). The research tried to examine proper position of the patches in order for the plate to achieve agreeable excitation modes. Intelligent structures are employed as option in aerospace structures that often exhibit open-loop behaviour which differs significantly from pre-flight ground test measurements or analytic predictions. This research also highlighted the intelligent structures method considering a structure with many distributed actuators, sensors, and processor networks. The best position for actuators was found in the centre of plate excitation. It showed that excitation modes describe good dynamics behaviour when the plate is excited at this point.

Intelligent control use a ranges of soft-computing tools such as neural networks, fuzzy logic, pattern recognition, genetic algorithms, particle swarm optimisation (PSO), simulated annealing, ant colony, aritificial immune system (AIS) and other metaheuristic optimisation algorithms. Normally the implementation of a control algorithm in a system is straightforward to realise. In contrast, the heuristics part is time consuming to implement and evaluate. The intelligent algorithm used in this work specifically for modelling and control comprise model based technique and non-model based adaptive techniques.

1.2.1. Genetic algorithm

GA is based on Darwinian principle of natural selection which could be described with the syntax "survival of the fittest" and the fact that the fittest individuals as a dominator over the weaker one in a competition environment. GA is also inspired with Mendel's laws of genetics that describe the principles of transferring hereditary properties from parents to offspring. Optimal solution is simply the product of the artificial evolution. GA was introduced by Holland in 1975 (Holland, 1992). This optimisation method operates with the population of points (individuals). Each individual of the population represents one possible solution to the optimisation problem. A positive real value, called fitness is used for the evaluation of the individual, i.e. for mapping the search space into the set of positive real numbers, where the order is well defined. Fitness indicates how well the individual solves the optimisation problem.

In general, the GA operation starts with initialisation of individuals at random. After an initial population is randomly generated, the algorithm evolves from current population into next one through three operators which are selection, crossover and mutation. Following is an explanation for each operation.

First, a selection or reproduction process allows better individual to pass on their genes to the next generation. The selection operator evaluates survival of the fittest that is judged by an objective function. There are many different techniques used in reproduction, some are roulette-wheel selection (the likelihood of picking an individual is proportional to the individual's score), tournament selection (a number of individuals are picked using roulette wheel selection, then the best of these is/are chosen for mating) and rank selection (pick the best individual every time). The simplest reproduction technique is through roulette-wheel.

Second, after the fit individual has been chosen in the selection process, the crossover exchanges the genes between two selected individuals. The primary purpose of the crossover operator is to get genetic material from the previous generation to the subsequent generation. Two individuals are chosen from the population using the selection operator. The term gene, chromosome/individual and population is illustrated in Figure 1.1. A crossover site is preferred along the bit strings. The values of the two strings are exchanged up to this point. If C1=000000 and C2=111111 and the crossover point is 2 then C1'=110000 and C2'=001111. The two new offspring created from this mating are put into the next generation of the population. The crossover operator causes recombining portions of good individuals, likely to create better individuals.

Third, mutation in a GA causes small alterations at single points in an individual's code. Its purpose is to maintain diversity within the population and hold back premature convergence. Some examples of mutation operators are generative, swap mode, destructive and swap sequence. Combination of these operators can be used in the evolution. However, when only one or two operators are employed, this will affect the algorithm as follows

- Mutation alone induces a random walk through the search space
- Mutation and selection (without crossover) create a parallel, noise-tolerant, hill-climbing algorithm
- Using selection alone will tend to fill the population with copies of the best individuals from the population
- Using selection and crossover operators will tend to cause the algorithm to converge on a good but sub-optimal solution

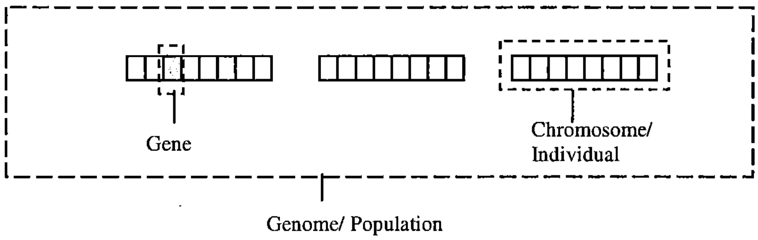

Figure 1.1: Gene, chromosome and genome

A number of advantages of GAs have been reported in the literature, mainly related to the engineering field. GA was used in solving the problems of multiobjective optimisation (Cvetkovic and Parmee, 1999) . Multi-objective GA (MOGA) is required when the problem involves trade-offs and no single optimal solution exists. GAs can also be 'black box' when they are able to solve problems without knowing about the problem from the start. As such, GAs do not require any derivatives of the objective function in order to calculate the optimum. Another advantage of these methods is that, unlike gradient methods, they are more likely to find global optima, and not be stuck on local optima, (Andersson, 2000). There are also shortcomings of GAs to take into consideration, for instance, a proper language presentation for input solutions and large number of evaluations of the objective function is required. Moreover, the fitness functions do not necessarily find the exact global optimum that cause a species to evolve into an evolutionary dead end.

1.2.2. Particle swarm optimisation

Global optimisation aims to find a solution in a solution set of objective function at determining not just "a local minimum" but also "the smallest local minimum" with respect to the solution set. There are two main categories of Global optimisation (GO) namely deterministic and probabilistic methods. The deterministic methods are concerned with heuristic application such as trajectory methods or penalty-based methods in reason to escape from local minima. Conversely, probabilistic methods rely on the probabilistic judgements to determine whether or not the search should depart from the neighbourhood of a local minimum. Evolutionary computation (EC) is member of stochastic search algorithm and has become a popular technique. EC employs a set of population and detects the optimal problem solution based on the evolution of biological life in the natural world. Algorithms based on EC include GA, genetic programming, evolutionary programming and artificial life methods.

EC can also be realised using particle swarm optimisation (PSO), which is one of the swarm intelligence techniques. PSO was proposed by Kennedy and Eberhart in 1995 based on the analogy of bird flocking (Reynolds, 1987) and fish school (Kennedy, 1995). PSO is related to artificial life and other EC techniques, especially GA in swarming theories. Like other EC techniques, PSO is a population-based search algorithm and is initialised with a population of random solutions, called particles. A 'particle' or 'agent' is a single solution in the search space like a 'bird' randomly searching food in an area, for instance. Birds do not know where the food is but they know how far the food is and their peers' positions. Thus an effective strategy is to follow the bird what is the nearest to the food, (Shi, 2001). In other words, each particle searches for optima in the problem space at every iteration. In every iteration, each particle changes its velocity towards its *pbest-* location of the best solution or personal best and *gbest-* location of the global best of particle achieved so far. Conceptually pbest resembles each individual experience and gbest is similar to publicised experience. Namely, each particle adjusting its 'flying' according to its own flying experiences as well as the flying experiences of other particles.

PSO basically developed through the concept of bird flocking simulation in two-dimensional space, has now been extended to N-dimensional. The position of each particle is represented by xy position coordinates and the velocity by *Vx* along x-axis and *Vy* along y-axis. Each particle tries to modify its position using four items of information. (i) the current position (x,y), (ii) the current velocities (*Vx*, *Vy*), (iii) the distance between the current position and pbest, (iv) the distance between the current position and gbest.

With the development and application of PSO algorithm in recent years, researchers have realised that PSO has several advantages. PSO comprises a very simple concept, and can be implemented in a few lines of computer code. It requires only simple mathematical operators; therefore, the whole calculation procedure of PSO is relatively simple and easy to be implemented. PSO is also computationally not expensive in terms of both memory requirements and speed. There are few parameters that PSO requires tuning (when incorporated with constriction factor) hence only the experiential settings is employed to ensure convergence. PSO has faster convergence rate than GA in most cases due to a single-directional information-sharing way, which allows PSO to approach the neighbourhood of optimal solution quickly. PSO can be applied to continuous optimisation problems and also extended to combinatorial problems in both discrete and continuous variables, (Eberhart and Shi, 2000).

Generally, realisations applied in evolutionary techniques are good at the application of PSO. Many authors embedded PSO into others algorithms for training purposes such as within artificial neural network (Su et al., 2007), ANFIS (Ghomsheh et al., 2007) and fuzzy neural network (Chatterjee et al., 2005). Furthermore, PSO is capable in solving various parameter estimation and optimisation problems. For example, PSO applied to optimise parameter settings of power system stabilisers and optimal flow (Abido, 2002).

1.2.3. Artificial immune system algorithm

Artificial immune system (AIS) is inspired by natural immune system composed of data manipulation, classification, and reasoning and representation methodologies. AIS are applied to problem solving and generally for the solution of real-world problems (de Castro and Timmis, 2002a). Natural immune system apparently follows a biological paradigm of the human immune system, meanwhile AIS is described theoretically as adaptive immune system by observing immune functions, principles and models. These combinations have lead to developed of AIS itself (de Castro and Timmis, 2002b).

The immune system is defined as the one responsible to protect the body against the attack from external microorganisms, (Tizard, 1995). Pathogen is an organism that produces diseases and is composed of antigen on the surface that refers to protein of bacterium, fungus or virus. An antigen is recognised by our bodies as strange substance and triggers the immune system into producing antibodies specific to that antigen. This means that if the same antigen is introduced in the upcoming, our immune system will recognise, remember and produce the right antibodies to battle with the intruders. There are two groups in the immune system: innate and adaptive immune system. The innate immune system is purposely for battle against bacterial infection while adaptive immune system is for antibody production specific to a determined infectious agent.

In white blood cells, lymphocytes are categories comprising T cells and B cells. Lymphocytes play an important and integral part of the body's defences that carry antigen receptors on their surfaces. The selected cell receptors are experienced to fine-tune the selective antigens. However, adaptive immune system can be positively and negatively regulated by the innate immunity. AIS is extensively used in many applications like pattern recognition, function approximation, data analysis and clustering, machine learning, associative memories, diversity generation and maintenance, evolutionary computation and programming, control and scheduling, computer and network security and generation of emergent behaviours. Some features of intelligent techniques used in this work are summarised in Table 1.1. Their particular advantages as optimisers to produce solution to modelling and controller estimation for later AVC strategies in this work are identified.

Table 1.1: General comparison between intelligent techniques used: GA, PSO and AIS algorithms

	Genetic Algorithms	Particle Swarm Optimisation Algorithm	Artificial Immune System Algorithm
Algorithm acronym /other types	***GA*** ES, EC, EP, GP	***PSO*** Tabu Search, Hill Climbing,	***AIS*** clonal selection, immune network, negative selection, cytokines selection
Population	chromosomes	Particles/ Cooperation agents	Antibodies/ cells
Fitness evaluated in a generation	Entire population	Each individual (local neighbourhood)	Entire population
Process	Reproduce, select the fittest, breed	Particles exist and adapt	Repertoire, proliferate, clone higher affinity and mutate less, replace lower affinity cell randomly
Fittest result	Member of final population	Member of final population	Entire final population of detectors

1.3 Active vibration control techniques

Methods of vibration control of flexible structures can be categorised into passive control and active control. In passive vibration control, passive elements, i.e, masses, dampers and/or springs, are applied to adjust the characteristics of controlled structures to the targeted design value. Passive control is inefficient for low frequencies and in most cases it can only control a certain limit of the structure response. On the other hand, active vibration control requires energy supplied to suppress the vibration.

The vibration problems in active control can be classified into two categories (McKinnell, 1989). First, active control uses controller-driven force to achieve modification to its global vibration level. The technique applies feedback control to add stiffness (using position feedback) and damping factor (via velocity feedback) to the system. The implementation in the feedback system may differ in applications illustrated in past literature work. The performance of the system is normally observed through global displacements or velocities. Second, active vibration control by means of isolation covering the motion in a certain region. Isolation is appropriate to be implemented when vibration is restricted to within susceptible region. The technique involves either applying feedback active elements to separate between the structure and its vibrating base, or use knowledge of disturbance response itself to generate control forces to cancel the effects of that signal. The latter is more effective since it can be implemented adaptively due to periodic nature of the excitation. This method was foremost used by Lueg in 1936 (Lueg, 1936) and found in active noise control applications (Leitch and Tokhi, 1987).

Active vibration control methods have been developed to circumvent many of the problems associated with structural vibration. A controlled single-link flexible manipulator using a time delayed feedback signal control was investigated in (Jnifene, 2007). A flexible appendage of spacecraft was modelled by a symmetrical rigid hub and cantilever beam and controlled with integrated input shaping-sliding mode output feedback control techniques in (Qing-lei et al., 2008). Sahin et al. (2008), modelled and controlled a smart aerospace structures was by means of finite element method (FEM) and spatial control involving aerodynamic loading. In particular study of active vibration control of a flat plate system was chosen as an application example. Uzal and Korbahti (2009) presented an exact analytical solution and demonstrated that it is possible to affect the eigenfrequencies and mode shapes of a plate by measuring the displacement and applying a pressure at discrete points on the plate. Da Silva et al. (2006) designed a robust controller for active damping using an linear matrix inqualities (LMI) framework, and a reduced model with observation and control spillover effects. The effectiveness of the approach showed that the damping was increased in some of the modes. Based on the concept of vibration cancellation, Rastgaar Aagaah et al. (2009) proposed an output feedback control, the orthogonal eigenstructure control (OEC), to find the control gains that decouple the modes of vibration and reduce transferring of vibration energy in a plate. The aforementioned studies are differing from one another in methodology used, but with similar intention in reducing vibration by applying external vibration energy.

Experiment based on active vibration control of flexible structures was conducted by using piezoelectric materials (PZT, lead zirconate titanate, etc.) as sensors/actuators (Bueno et al., 2008; Sahin et al., 2008; Shimon and Hurmuzlu, 2007). Active vibration control based on intelligent algorithms began several decades ago with use of neural network and fuzzy logic in modelling and adaptive control of systems (Meziane and Benalla, 2007; Phan and Gale, 2008; Ratneshwar and Chengli, 2002; Zhao and Virvalo, 1993). Furthermore, intelligent algorithms have been used as optimisation method to find actuator and sensor location where the optimal suppression level can be achieved (Mehrabian and Yousefi-Koma, 2007; Roseiro et al., 2006).

More recently, the research topic of active vibration control flexible structures has received considerable attention. Mat Darus (2004) investigated parametric and non-parametric modelling approaches using RLS, GAs, NNs and fuzzy logic and utilised them to control vibration in flexible plate structures within single-input single-output (SISO) and single-input multi-output (SIMO) controlled structure. Mohd Hashim (2004) extended the work considering plate and beam structure using direct control approach in SISO system.

This work presented in thesis incorporates conventional RLS-based algorithms and intelligent algorithms comprising real coded GA, PSO and AIS independently in modelling and control of flexible plate when subjected to various external excitations of PRBS, random and finite duration step types. Investigations into non-model based vibration control are also carried out. The approaches are realised in both SISO and SIMO control configurations.

1.4 Flexible plate system

A structure can be defined as flexible once it exhibits considerably low damping ratio and long time vibration. This includes beams, flexible-joint manipulators, bar-linkage mechanism, plate, trusses and other shapes of such structures. Among them, a plate is defined as a smooth, flat, relatively thin, rigid body of uniform thickness. The structure is relatively large-scale and will vibrate within lower natural frequency. Thus, in such cases, the effort to control all modes of the structure will be wastage of energy.

The stipulation of operation requires flexible structures to be light in weight. Moreover the low cost of transportation will increase the functionality, in particular for space structure area. The space application includes structure based of radar antenna, solar panel, space robotic and the like. A flexible structure has infinite number of modes with low flexible rigidity and material damping. Thus, a flexible structure may vibrate in large-amplitude vibration and continuous long time when subjected to a small excitation. In order to determine the characteristics of a plate structure, a rigorous analysis can be carried out using various approaches to obtain its governing dynamic equations. The approaches include Mindlin-type shear deformation theory for the first order of shear type deformation, Timoshenko theory for an isotropic homogeneous plate (Timoshenko and Woinowsky-Krieger, 1959) and Levy-type solutions for harmonic of the applied load or edge moments using the principle of virtual work.

In the present study, a clamped thin plate model is used to describe the system dynamics and it is validated by comparing the natural frequencies with the corresponding theoretical values from (Leissa, 1969). A relevant study was reported by Mat Darus (2004) for analysis of flexible plate structure. A state space formulation with finite difference discretisation for vibration control of a flexible plate system has not been reported in the literature. In this work, PRBS, random and finite duration step excitations are used to assess the applicability of the AVC technique for vibration suppression in flexible plate systems.

1.5 Aim and objectives of the research

The main focus of work presented in this thesis is development of AVC approaches for flexible plate structures using biologically- inspired optimisation approaches comprising GA, PSO and AIS. The main objectives of the work are:

i. To study and analyse the dynamic behaviour of flexible plates in different configurations
ii. To develop parametric models of the plate using RLS, GA, PSO and AIS algorithms.
iii. To develop and realise model-based AVC system for the plate within SISO and SIMO control structures
iv. To develop and realise non-model based AVC systems for the plate within SISO and SIMO control structures

Investigations are carried out on analysing a square plate with different boundary conditions. The central finite difference (FD) method is used to discretise the governing dynamic equation of the plate thus is transformed into a state-space formulation. A simulation algorithm characterising the behaviour of the plate is thus developed as a testbed for analysis and control development. Four different plate configurations namely all edges clamped, one edge clamped, two edges clamped and three edges clamped are considered. The solution of governing by FD method and state space formulation has previously been adopted for flexible manipulators (Azad, 1994) and cantilever beams (Hossain, 1995). In (Azad, 1994), finite difference is utilised in solving the governing partial differential equations and the developed algorithm is implemented using the state space formulation. Their solution into state-space formulation is simpler due to diagonal elements in the matrix formulation. However for plates a truncated matrix is required which is complicated with substitution of boundary condition.

1.6 Thesis outline

A brief outline of the thesis contents is given as follows:

Chapter 1 briefly describes the intelligent algorithms and AVC techniques. An introduction to flexible plate system is given. Moreover, objectives of the research and the contributions are presented. The publications arising from the work is listed at the end of the chapter.

Chapter 2 describes dynamic modelling of a flexible plate using the FD method and state space formulation. The simulation algorithm characterising the dynamic behaviour of the structure is implemented within the Matlab/Simulink environment, and its performance of modelling been assessed in time and frequency domains. The simulation environment thus developed is used in subsequent research work as a platform for development and verification of suitable control strategies for vibration suppression.

Chapter 3 presents the development of parametric models using conventional recursive least square (RLS) and intelligent GA, PSO and AIS algorithms. The models are validated using time- and frequency-domain response, mean squares of error and correlation tests. The performance of the algorithm are assessed through determination of dominant vibration modes using spectral density of the response.

Chapter 4 presents the development of AVC systems utilising RLS-, GA-, PSO- and AIS-model based approaches within SISO control structure. The AVC development is carried out utilising PRBS, random and finite duration step disturbance signals. A comparative assessment of performance of the SISO-AVC model based technique is presented based on vibration reduction at dominant modes of vibration of the system.

Chapter 5 presents the development of AVC systems utilising RLS-, GA-, PSO- and AIS-model based approaches within SIMO control structure. The AVC development is carried out utilising a single input-two output system subjected to the three different disturbance signals. The performance of the SIMO-AVC model based technique is assessed based on vibration reduction at the dominants modes of the plate.

Chapter 6 presents the development of a non-model based AVC system approach using GA, PSO and AIS within SISO and SIMO control structure. The AVC development is considered with one input-one output and one input-two output system configurations subjected to the three different disturbance signals. Comparative performance assessments of non model based SISO- and SIMO-AVC and the model based SISO- and SIMO-AVC are presented based on the vibration reduction at dominant modes of the plate.

Chapter 7 presents summary and conclusions of the work and highlights areas for further research.

1.7 Thesis contribution

The contributions of the research can be highlighted as follows:

In this study, a dynamic model of a flexible plate is developed using finite difference method and state space formulation. The simulated plant is able to characterise the dynamic behaviour of the structure by identifying the resonance frequency of the structure. This work focused on the first five modes of resonance frequency since their greatest impact on the amplitude of motion. The state space application offer faster computational time and less complexity effort. This new dynamic model implemented within Matlab/SIMULINK is a novel approach to perform the AVC controller strategy.

Subsequently, model based and non model based AVC controller approach is implemented in this work using SISO and SIMO configuration. Implementation of modelling estimation from the simulated plant is able to perform by designing an ON and OFF phase of secondary source(s). Meanwhile non model based AVC controller is carried out to perform the simplified controller by bypass the model estimation. Such controller promises fair or better performance in terms of attenuation achieved for the first five modes. Both controller strategy developed in this work has not been addressed before for such flexible plate structure developed within the aboved configurations.

The use of intelligent algorithm in optimisation of standard function is received widely attention in currently study. This includes modification in many ways of coding process, adaptation of objective function and convergence of the algorithm towards better performance. This work is differ when intelligence feature is utilised to optimise the controller characteristic targeting at zero deflection of observation point. In this work extended GA of real values is used to optimise the mean squared error of the output response before and after controlled. Similarly, PSO with spreading factor is implemented for this application of controller. It is proved that real value of GA and faster convergence of PSO capable in well vibration suppression.

In addition, AIS based on Clonalg and AINET algorithm to the authors knowledge is firstly applied in optimising such mse function of the state space structure response. This is correspondingly used to characterise the AVC controller functions. In terms of computational time, less operator used in AiNet give benefits in simulation and implementation for the future practice.

1.8 Publications

Part of the work presented in the thesis is either printed or submitted and to be appearing as follows:

Journal papers:

SALLEH, S.M. and Tokhi, M.O.: Discrete simulation of flexible plate structure using state-space formulation. Journal of System Simulation, 20 (19): p. 5141-5146 (2008).

Conference papers:

SALLEH, S. M., TOKHI, M. O. and MOHAMAD, M. (2009). Modelling of a flexible plate using RLS with variable and directional forgetting factor. *Proceedings of ICM2009: 2009 IEEE International Conference on Mechatronics*, Malaga, Spain, 14 – 17 April 2009.

SALLEH, S. M. and TOKHI, M. O. (2009). Active vibration control of a flexible plate using recursive least square with directional forgetting factor. *Proceedings of ICSV16: Sixteenth International Congress on Sound and Vibration*, Krakow, Poland, 05 – 09 July 2009.

SALLEH, S. M., TOKHI, M. O. and TOHA, S. F. (2009). Real-coded genetic algorithm identification of a flexible plate system. *Proceedings of ICINCO-2009: Sixth International Conference on Informatics in Control, Automation and Robotics*, SPSMC, Milan, Italy, 02 – 05 July 2009.

SALLEH, S. M. and TOKHI, M. O. (2009). Parametric modelling of a flexible plate structure using artificial immune system algorithm. In Andrewa, P. S., Timmis, J., Owens, N. D. L., Aickelin, U., Hart, E., Hone, A., and Tyrrel, A. M. (editors), *Proceedings of ICARIS 2009: 8th International Conference on Artificial Immune Systems*, York, UK, 09 – 12 August 2009, pp. 301–314. Springer-Verlag, Berlin Heidelberg.

SALLEH, S. M. and TOKHI, M. O. (2009). Self-tuning AIS-based active vibration control of a thin plate. *Proceedings of the 2009 8th IEEE International Conference on Cybernetic Intelligent Systems*, Birmingham, UK, 9-10 September, 2009, pp. 129–134.

SALLEH, S. M. and TOKHI, M. O. (2009). Active vibration control of a flexible plate using real coded genetic algorithm. *Proceedings of ICONS 2009: The 2nd IFAC International Conference on Intelligent control Systems and Signal Processing*, Istanbul, Turkey, 21 – 23 September 2009.

SALLEH, S. M. and TOKHI, M. O. (2009). Tuning of RLS-active vibration controller using genetic algorithm. *Proceedings of the 2009 8th IEEE International Conference on Cybernetic Intelligent Systems*, Birmingham, UK, 9-10 September, 2009, pp. 153–158.

SALLEH, S. M., TOKHI, M. O., JULAI, S., MOHAMMAD, M. and ABD LATIFF, I. (2009). PSO-based parametric modelling of a thin plate structure. *Proceedings of EMS2009: UKSim Third European Symposium on Computer Modelling and Simulation*, Athens, Greece, 25–27 November 2009, pp. 43–48.

SALLEH, S. M., TOKHI, M. O. & TOHA, S. F. Artificial immune network model based controller for vibration suppression of flexible structures. *Evolutionary Computation (CEC), Congress on IEEE 2010, Barcelona, Spain, July 2010.*

Chapter 2

Dynamic Characterisation and Simulation of a Flexible Plate System

2.1 Introduction

Flexible structures are extensively of engineering interest. The computation of natural frequencies needs to be accurate to represent the dynamic characteristic of the structure. Many methods have been used to represent the dynamic motion of plates, including the finite difference (FD) method, differential transformation method (DTM), finite element (FE) method, and the Galerkin and Ritz method. Yeh et al. (2006) used a hybrid of FD and DTM as numerical tool that combined Taylor series expansion in the form of a polynomial solution, and separated coordinates of state variable and solved the initial values problems concurrently. Talebian et al. (2010) solved the governing linear eigenvalue partial differential equation using a Galerkin method in order to reduce the model order and determine the natural frequencies of a microplate. Guennam and Luccioni (2009) compared the numerical models of piezoactuated beam using two modelling approaches; brick and shell FE types. The FE techniques have introduced great improvement upon previous methods. Nevertheless, most of the FE approaches use integration of eigenvectors to obtain the bending stresses that cause loss in accuracy and is time consuming. Combination of state space and FE method give advantage to reduce the final algebraic equation thus reducing computer effort. Foremost advantages of using the state space formulation are reduction in the number of coefficients, easing complex computation and reduction in the hardware cost. For these reasons, a state space formulation is adopted in this work.

In this chapter the development of plate model refers to solving the governing dynamic equation. The FD method is used to discretise the governing dynamic equation considered with no damping and the lateral deflection of plate is obtained using central FD method. Four types of boundary conditions namely all clamped edges; three, two and one clamped edge are considered. The resulting representation is transformed into discrete state-space equation. The general solution is thus obtained and used to simulate the dynamic behaviour of the plate in the Matlab/Simulink environment.

2.2 The flexible plate system

In the classical theory of plates, the equation of motion is derived from virtual displacements from geometric and boundary conditions. The plate is defined as structural element where its thickness is smaller in dimension than its width and length. A thin square plate of length *a*, along x-axis, width, *b*, along y-axis and thickness, *h* along z-axis is considered with shear forces per unit length, Q_x , Q_y and twisting moments using 'right-hand screw' rule, as illustrated in Figures 2.1 and 2.2.

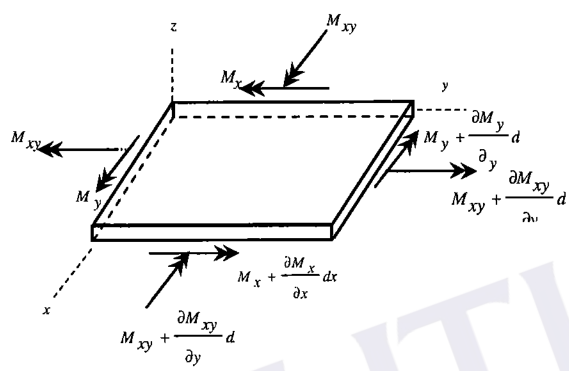

Figure 2.1: Moments of the plate element

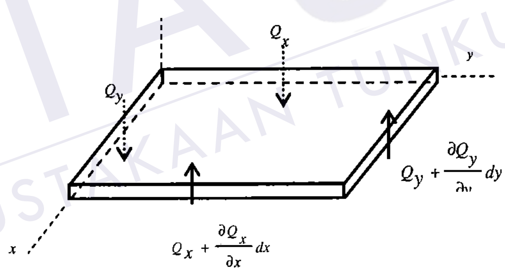

Figure 2.2: Shear forces of the plate element

From the bending and twisting moments acting on the plate due to the intensity of a load *q*, the plate is assumed to experience a small deflection, w.The earliest work on clamped edges with small deflection was reported by Samuel (1942). Their result using normal pressure was close to the result obtained in 1913 by Hencky. A theoretical analysis was given for the stresses and deflections of a square plate with clamped edges under normal pressure producing large deflections. Values of the bending stress and membrane stress at the centre of the plate and at the midpoint of the edge were given for centre deflections up to 1.9 times the plate thickness (Samuel, 1942).

Summing all the forces in the z direction and considering the effect of shear forces Q_x and Q_y in terms bending moments, M_x, M_y and M_{xy} yield (Timoshenko, 1959) :

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{\rho}{D}\frac{\partial^2 w}{\partial t^2} = \frac{q}{D} \tag{2.1}$$

where w represents lateral deflection in the z direction, ρ is mass density per unit area, q is transverse external force ($q(x,y)=$ force per unit area), $\partial^2 w/\partial t^2$ is acceleration in the z direction, $D = Eh^3/12(1-v^2)$ is the flexural rigidity, v is Poisson ratio, h is thickness of the plate and E is Young Modulus. Equation (2.1) represents the dynamic equation characterising the behaviour of the flexible plate.

2.2.1 Finite difference representation

In this section solution of the governing dynamic equation of the plate in equation (2.1) is obtained using the FD method. Finite difference is a method of approximating a differential operator to solve differential equations. In numerical analysis, this method is used in ordinary differential equations and partial differential equations (PDEs) by replacing the derivatives in those equations by finite differences. In this case (Figure 2.3), the computation domain is divided into small regions and each region is assigned with reference indeces i and j. Each of the interior grid represents $x_i = i\Delta x$, $y_j = j\Delta y$ or regions with $i = 0,1,2,\ldots,n$; $j = 0,1,2,\ldots,m$.

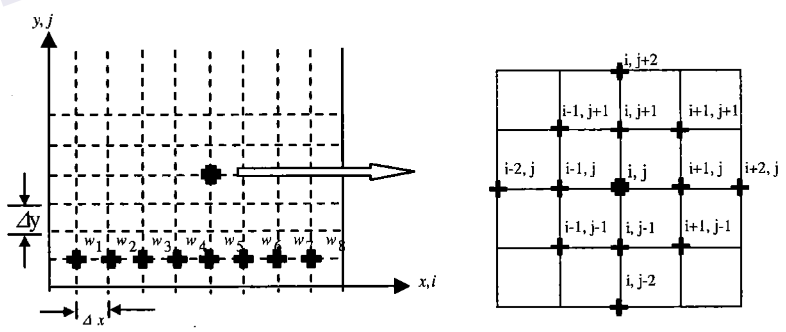

Figure 2.3: Finite discretisation of coordinates i,j

The third coordinate represents the time with reference k, where $t = k\Delta t$, $k = 0,1,2,\ldots, p$. This can be illustrated in Figure 2.4 as the small region moves along horizontal coordinates.

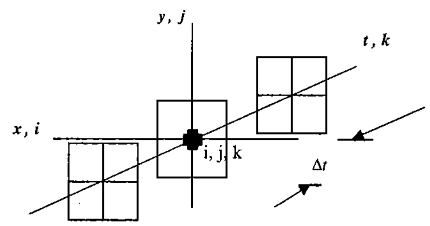

Figure 2.4: A coordinate of three dimensional flexible plate

Using Taylor expansion series, the central FD formulae are calculated for the partial derivative terms of the response (deflection), $w(x,y,t) = w_{i,j,k}$ of the plate at point $x = i\Delta x$, $y = j\Delta y$ and $t = k\Delta t$. Therefore, using central FD approximations, the solution of PDE for the partial derivative terms of $w(x,y,t)$ can be expressed as

$$\left(\frac{\partial^2 w}{\partial t^2}\right)_{i,j,k} = \frac{w_{i,j,k+1} - 2w_{i,j,k} + w_{i,j,k-1}}{\Delta t^2} \tag{2.2}$$

$$\left(\frac{\partial^3 w}{\partial x^3}\right)_{i,j,k} = \frac{w_{i+2,j,k} - 2w_{i+1,j,k} + 2w_{i-1,j,k} - w_{i-2,j,k}}{2\Delta x^3} \tag{2.3}$$

$$\left(\frac{\partial^3 w}{\partial y^2 \partial x}\right)_{i,j,k} = \frac{\begin{pmatrix} w_{i+1,j+1,k} + 4w_{i-1,j+1,k} - 6w_{i+1,j,k} \\ + w_{i-1,j,k} - w_{i+1,j-1,k} + w_{i-1,j-1,k} \end{pmatrix}}{2\Delta y^2 \Delta x} \tag{2.4}$$

$$\left(\frac{\partial^4 w}{\partial x^4}\right)_{i,j,k} = \frac{w_{i+2,j,k} - 4w_{i+1,j,k} + 6w_{i,j,k} - 4w_{i-1,j,k} + w_{i-2,j,k}}{\Delta x^4} \tag{2.5}$$

$$\left(\frac{\partial^4 w}{\partial y^4}\right)_{i,j,k} = \frac{w_{i,j+2,k} - 4w_{i,j+1,k} + 6w_{i,j,k} - 4w_{i,j-1,k} + w_{i,j-2,k}}{\Delta y^4} \tag{2.6}$$

$$\left(\frac{\partial^4 w}{\partial x^2 \partial y^2}\right)_{i,j,k} = \frac{\begin{pmatrix} 4w_{i,j,k} - 2(w_{i+1,j,k} + w_{i,j+1,k} + w_{i-1,j,k} + w_{i,j-1,k}) \\ + w_{i+1,j+1,k} + w_{i-1,j+1,k} + w_{i-1,j-1,k} + w_{i+1,j-1,k} \end{pmatrix}}{\Delta x^2 \Delta y^2} \tag{2.7}$$

Substituting equations (2.2) and (2.5) – (2.7) into equation (2.1) and rearranging the solution can be written as (Mat Darus, 2004)

$$
\begin{aligned}
w_{i,j,k+1} = -\frac{D\Delta t^2}{\rho}(P w_{i,j,k} &+ Q(w_{i+1,j,k} + w_{i-1,j,k}) \\
&+ R(w_{i,j+1,k} + w_{i,j-1,k}) + S(w_{i+1,j+1,k} + w_{i-1,j+1,k} \\
&+ w_{i-1,j-1,k} + w_{i+1,j-1,k}) + T(w_{i+2,j,k} + w_{i-2,j,k}) \\
&+ U(w_{i,j+2,k} + w_{i,j-2,k}) \\
&+ 2w_{i,j,k} - w_{i,j,k-1} + \frac{\Delta t^2 q_{i,j}}{\rho}
\end{aligned} \tag{2.8}
$$

where

$$
P = \frac{6}{\Delta x^4} + \frac{8}{\Delta x^2 \Delta y^2} + \frac{6}{\Delta y^4}, Q = -\frac{4}{\Delta x^4} - \frac{4}{\Delta x^2 \Delta y^2},
$$

$$
R = -\frac{4}{\Delta y^4} - \frac{4}{\Delta x^2 \Delta y^2}, S = \frac{2}{\Delta x^2 \Delta y^2}, T = \frac{1}{\Delta x^4}, U = \frac{1}{\Delta y^4}
$$

Dividing the plate into equal-length sections along x and y; i.e. $\Delta x = \Delta y = \Delta_{xy}$, equation (2.8) can be simplified as (Mat Darus, 2004) :

$$
\begin{aligned}
w_{i,j,k+1} = -\frac{D\Delta t^2}{\rho \Delta_{xy}^{\ 4}} * \\
\begin{pmatrix} 20 w_{i,j,k} - 8(w_{i+1,j,k} + w_{i,j+1,k} + w_{i-1,j,k} + w_{i,j-1,k}) \\
\quad + 2(w_{i+1,j+1,k} + w_{i-1,j+1,k} + w_{i-1,j-1,k} + w_{i+1,j-1,k}) \\
\quad + w_{i+2,j,k} + w_{i-2,j,k} + w_{i,j+2,k} + w_{i,j-2,k} \end{pmatrix} \\
+ 2w_{i,j,k} - w_{i,j,k-1} + \frac{\Delta t^2 q_{i,j}}{\rho}
\end{aligned} \tag{2.9}
$$

Equation (2.9) gives the general solution of PDE using the FD method. This equation will be used to obtain discrete state-space formulation reflecting the state vector and matrix of the system with the boundary conditions. Two types of boundary conditions will be considered with the initial condition of the plate as zero deflection. In this case, the forces and moments of the plate due to its weight are neglected (Mat Darus, 2004). Thus, $w_{i,j,k}\big|_{t=0} = 0$ and the boundary conditions to be considered are:

i) Clamped Edge: No deflection occurs at all times if the edge is clamped and the tangent of the deflection is also zero (Timoshenko, 1959). By applying the FD approximation with Taylor expansion to,

$$w\big|_{y=a} = \frac{\partial w}{\partial y}\bigg|_{y=a} = 0 \tag{2.10}$$

the associated boundary conditions are transformed into

$$\begin{aligned} w_{i,j,k} &= 0 \\ w_{i,j+1,k} &= w_{i,j-1,k} \end{aligned} \tag{2.11}$$

ii) A Free Edge: For a free edge, it is natural to assume no bending and twisting moments along the edge as well as no vertical shear forces. Thus, considering the edge is free along y = *a*, then

$$M_x\big|_{x=a} = M_{xy}\big|_{x=a} = Q_x\big|_{x=a} = 0 \tag{2.12}$$

These three boundary conditions of free edge have been reduced into two by Kelvin and Tait (Timoshenko, 1959). Thus substituting the relevant expressions for M_x, M_{xy}, Q_x in terms of w, equation (2.12) is simplified as

$$\begin{aligned} w_{i,j,k} = {} & \frac{1}{2+2v}(w_{i+1,j,k} + w_{i-1,j,k}) + \frac{v}{2+2v}(w_{i,j+1,k} + w_{i,j-1,k}) \\ & - \frac{v}{(4+4v)\Delta_{xy}}(w_{i+2,j,k} - 2w_{i+1,j,k} + 2w_{i-1,j,k} - w_{i-2,j,k}) \\ & - \frac{2-v}{(4+4v)\Delta_{xy}}(w_{i+1,j+1,k} + w_{i-1,j+1,k} - 2w_{i+1,j,k} \\ & + 2w_{i-1,j,k} + w_{i+1 j-1,k} - w_{i-1,j-1,k}) \end{aligned} \tag{2.13}$$

As central FD requires fictitious points out of bounded region, to obtain deflection of the bounded region, the fictitious deflections must be avoided. The known boundary conditions related to the dynamic equation of the plate are utilised to eliminate these fictitious points (Mat Darus, 2004).

2.2.2 State Space formulation

Using matrix notation, equation (2.9) can be written in a compact form as

$$\mathbf{W}_{i,j,k+1} = A\mathbf{W}_{i,j,k} + 2\mathbf{W}_{i,j,k} - \mathbf{W}_{i,j,k-1} + C\mathbf{F} \tag{2.14}$$

Rearranging the above gives:

$$\mathbf{W}_{i,j,k+1} = (A + 2_{ijk})\mathbf{W}_{i,j,k} - \mathbf{W}_{i,j,k-1} + C\mathbf{F} \tag{2.15}$$

where 2_{ijk} represents the diagonal elements of $\left(\frac{2}{c}\right)$ in $\mathbf{W}_{i,j,k}$ and is added into matrix A with,

$$\mathbf{W}_{i,j,k+1} = \begin{bmatrix} w_{1,1,k+1} \\ w_{1,2,k+1} \\ \vdots \\ w_{1,m+1,k+1} \\ w_{2,1,k+1} \\ w_{2,2,k+1} \\ \vdots \\ w_{2,m+1,k+1} \\ w_{3,1,k+1} \\ w_{3,2,k+1} \\ \vdots \\ w_{n+1,m+1,k+1} \end{bmatrix}, \mathbf{W}_{i,j,k} = \begin{bmatrix} w_{1,1,k} \\ w_{1,2,k} \\ \vdots \\ w_{1,m+1,k} \\ w_{2,1,k} \\ w_{2,2,k} \\ \vdots \\ w_{2,m+1,k} \\ w_{3,1,k} \\ w_{3,2,k} \\ \vdots \\ w_{n+1,m+1,k} \end{bmatrix}, \mathbf{W}_{i,j,k-1} = \begin{bmatrix} w_{1,1,k-1} \\ w_{1,2,k-1} \\ \vdots \\ w_{1,m+1,k-1} \\ w_{2,1,k-1} \\ w_{2,2,k-1} \\ \vdots \\ w_{2,m+1,k-1} \\ w_{3,1,k-1} \\ w_{3,2,k-1} \\ \vdots \\ w_{n+1,m+1,k-1} \end{bmatrix}, \mathbf{F} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ f_{i,j} \\ 0 \\ \vdots \\ o \end{bmatrix}$$

where $C = \left(\frac{\Delta t^2}{\rho}\right), c = -D\frac{\Delta t^2}{\rho}$ and

$$
A = \begin{bmatrix}
a_1 & a_2 & a_3 & 0 & \cdots & 0 & a_4 & a_5 & a_4 & 0 & \cdots & 0 & a_6 & 0 & 0 \\
a_2 & a_1 & a_2 & a_3 & 0 & \cdots & 0 & a_4 & a_5 & a_4 & 0 & \cdots & 0 & \ddots & 0 \\
a_3 & a_2 & a_1 & a_2 & a_3 & 0 & \ldots & 0 & a_4 & a_5 & a_4 & 0 & \ddots & \ddots & a_6 \\
0 & a_3 & a_2 & a_1 & a_2 & a_3 & 0 & \ldots & 0 & a_4 & a_5 & \ddots & \ddots & \cdots & 0 \\
\vdots & 0 & a_3 & a_2 & a_1 & a_2 & a_3 & 0 & \ldots & 0 & \ddots & \ddots & a_4 & 0 & \vdots \\
0 & \ldots & 0 & a_3 & a_2 & a_1 & a_2 & a_3 & 0 & \ddots & \ddots & a_4 & a_5 & a_4 & 0 \\
a_4 & 0 & \ldots & 0 & a_3 & a_2 & a_1 & a_2 & \ddots & \ddots & \ldots & 0 & a_4 & a_5 & a_4 \\
a_5 & a_4 & 0 & \ldots & 0 & a_3 & a_2 & \ddots & \ddots & a_3 & 0 & \ldots & 0 & a_4 & a_5 \\
a_4 & a_5 & a_4 & 0 & \ldots & 0 & \ddots & \ddots & a_1 & a_2 & a_3 & 0 & \ldots & 0 & a_4 \\
0 & a_4 & a_5 & a_4 & 0 & \ddots & \ddots & a_3 & a_2 & a_1 & a_2 & a_3 & 0 & \ldots & 0 \\
\vdots & 0 & a_4 & a_5 & \ddots & \ddots & \ldots & 0 & a_3 & a_2 & a_1 & a_2 & a_3 & 0 & \ldots \\
0 & \cdots & 0 & a_4 & a_5 & a_4 & 0 & \ldots & 0 & a_3 & a_2 & a_1 & a_2 & a_3 & 0 \\
a_6 & 0 & \cdots & 0 & a_4 & a_5 & a_4 & 0 & \ldots & 0 & a_3 & a_2 & a_1 & a_2 & a_3 \\
0 & a_6 & 0 & \cdots & 0 & a_4 & a_5 & a_4 & 0 & \ldots & 0 & a_3 & a_2 & a_1 & a_2 \\
0 & 0 & a_6 & 0 & \cdots & 0 & a_4 & a_5 & a_4 & 0 & \ldots & 0 & a_3 & a_2 & a_1
\end{bmatrix}
$$

$W_{i,j,k+1}$ is the deflection of grid points $i = 1, 2, \ldots\ldots, n+1$ and $j = 1, 2, \ldots, m+1$ at time step k+1. $W_{i,j,k}$ and $W_{i,j,k-1}$ are the corresponding deflections at time steps k and k-1 respectively. A is constant $(n+1)(m+1)\times(n+1)(m+1)$ matrix whose entries depend on physical dimensions and characteristics of the plate, B is a diagonal matrix of -1 corresponding to $W_{i,j,k}$ and C is a scalar related to the given input and F is an $(n+1)(m+1)\times 1$ matrix known as the forcing matrix. where

$$
a_1 = Pc + \frac{2}{c}, a_2 = Qc, a_3 = Tc,
$$
$$
a_4 = Sc, a_5 = Rc, a_6 = Uc
$$

Also in state space equation matrices are denoted as boldface uppercase letters (***X***) and vectors are denoted by boldface lowercase letters (***x***). Scalar signals will be denoted by normal lowercase letters (x). A state space formulation can be generally constructed by referring to the matrix formulation with state dimension N, m inputs and p outputs as:

$$
\begin{aligned}
\boldsymbol{x}(n+1) &= \boldsymbol{P}\boldsymbol{x}(n) + \boldsymbol{Q}\boldsymbol{u}(n) \\
\boldsymbol{y}(n) &= \boldsymbol{R}\boldsymbol{x}(n) + \boldsymbol{S}\boldsymbol{u}(n)
\end{aligned}
\tag{2.16}
$$

where $u(n)$ is the input to the system, $\boldsymbol{x}(n)$ is the state of the system and $\boldsymbol{y}(n)$ is the output of the system. The elements of the above matrices can also be written as

REFERENCES

ABD LATIFF, I. & TOKHI, M. O. (2009) Fast convergence strategy for particle swarm optimization using spread factor. *Proceedings of the Eleventh conference on Congress on Evolutionary Computation.* Trondheim, Norway, IEEE Press.

ABIDO, M. A. (2002) Optimal design of power system stabilizers using particle swarm optimization. *IEEE Transactions on Energy Conversion,* 7, 406 - 413

ALAM, M. & TOKHI, M. (2007) Modelling of a twin rotor system: a particle swarm optimization approach. *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering,* 221, 353-374.

ALLEN, M. S. & GINSBERG, J. H. (2006) A global, single-input-multi-output (SIMO) implementation of the algorithm of mode isolation and application to analytical and experimental data. *Mechanical Systems and Signal Processing,* 20, 1090-1111.

ALTINTAS, G. & BAGCI, M. (2005) Determination of the steady-state response of viscoelastically supported rectangular orthotropic mass loaded plates by an energy-based finite difference method. *Journal of Vibration and Control,* 11, 1535-1552.

ANDERSSON, J. (2000) A survey of multiobjective optimization in engineering design. *Technical Report* Linkping, Sweden, Department of Mechanical Engineering, Linkping University.

ANTSAKLIS, P. J. (1999) Intelligent control. *Encyclopedia of Electrical and Electronics Engineering.* , ed., John Wiley & Sons, Inc.

ANTSAKLIS, P. J. & PASSINO, K. M. (1993) *An introduction to intelligent and autonomous control*, Kluwer Academic Publishers.

ATTALLAH, K. M. Z., YE, J. Q. & SHENG, H. Y. (2007) Three-dimensional finite strip analysis of laminated panels. *Computers & Structures,* 85, 1769-1781.

AZAD, A. K. M. (1994) *Analysis and design of control mechanisms for flexible manipulator systems. Dept. of Automatic Control And Systems Engineering.* Sheffield, UK, University of Sheffield.

BAILEY, T. & HUBBARD, J. E. (1985) Distributed piezoelectric polymer active vibration control of a cantilever beam. 8, 605.

BALAS, G. J. & DOYLE, J. C. (1990) Identification of flexible structures for robust control. *Control Systems Magazine, IEEE,* 10, 51-58.

BAZLEY, N.W., FOX, D.W. and Stadter, J.T. (1965). Upper and lower Bounds for frequencies of rectangular cantilever plates. Technical Memo TG-705, Applied Physics Lab., The John Hopkins University, USA

BERTIN, D., BITTANTI, S. & BOLZERN, P. (1985) Prediction-error directional forgetting technique for recursive estimation. *Systems Science,* 11, 33-39.

BILLINGS, S. A. & VOON, W. S. F. (1986) Correlation based model validity tests for non-linear models. *International Journal of Control,* 44, 235 - 244.

BILLINGS, S. A. & ZHU, Q. M. (1995) Model validation tests for multivariable nonlinear models including neural networks. *International Journal of Control,* 62, 749-766.

BITTANTI, S., BOLZERN, P. & CAMPI, M. (1990) Exponential convergence of a modified directional forgetting identification algorithm. *Systems and Control Letters,* 14, 131-137.

BUENO, D. D., MARQUI, C. R., SANTOS, R. B., NETO, C. M. & VICENTE LOPES, J. (2008) Experimental active vibration control in truss structures considering uncertainties in system parameters. *Mathematical Problems in Engineering,* vol. 2008, 14 pages.

BURNET, F. (1959) The clonal selection theory of acquired immunity. Cambridge University Press. London.

CAI, G.-P. & LIM, C. (2006) Optimal tracking control of a flexible hub–beam system with time delay. *Multibody System Dynamics,* 16, 331-350.

CHATTERJEE, A., PULASINGHE, K., WATANABE, K. & IZUMI, K. (2005) A particle-swarm-optimized fuzzy–neural network for voice-controlled robot systems. *IEEE Transaction on Industrial Electronics,* 52, 1478-1489.

CHEN, W. (2001) Dynamic modeling of multi-link flexible robotic manipulators. *Computers & Structures,* 79, 183-195.

CHIPPERFIELD, A. J., FLEMING, P. J., POHLHEIM, H. & FONSECA, C. M. (1994) A genetic algorithm toolbox for MATLAB. *International Conference on Systems Engineering,. Coventry, UK.*

CHOPRA, I. (2002) Review of state of art of smart structures and integrated systems. 40, 2145.

CHU, C.-L., WU, B.-S. & LIN, Y.-H. (2006) Active vibration control of a flexible beam mounted on an elastic base. *Finite Element Analysis and Design,* 43, 59-67.

CLARK, R. L. (1997) Accounting for out-of-bandwidth modes in the assumed modes approach: implications on colocated output feedback control. *Journal of Dynamic System, Measurement, and Control,* 119, 390.

CLARK, R. L. & FULLER, C. R. (1992) Experiments on active control of structurally radiated sound using multiple piezoceramic actuators. 91, 3313.

CVETKOVIC, D. & PARMEE, I. C. (1999) Genetic algorithm-based multi-objective optimisation and conceptual engineering design. *Proceedings of the 1999 Congress on Evolutionary Computation, 1999. CEC 99.*

DA SILVA, S., JUNIOR, V. L. & BRENNAN, M. J. (2006) Design of a Control System using Linear Matrix Inequalities for the Active Vibration Control of a Plate. *Journal of Intelligent Material Systems and Structures,* 17, 81-93.

DE CASTRO, L. & TIMMIS, J. (2002) An artificial immune network for multimodal function optimization. IN (ED.), R. E. (Ed.) *CEC'02, Part of the 2002 IEEE World Congress on Computational Intelligence.* Honolulu, HI, IEEE Piscataway.

DE CASTRO, L. & TIMMIS, J. (2002) *Artificial Immune Systems: A New Computational Intelligence Approach*, Springer.

DE GERSEM, H., MOENS, D., DESMET, W. & VANDEPITTE, D. (2005) A fuzzy finite element procedure for the calculation of uncertain frequency response